

## **Modelling the Security of Key Exchange**

Colin Boyd

including joint work with Janaka Alawatugoda,  
Juan Gonzalez Nieto

Department of Telematics, NTNU

Workshop on Tools and Techniques for Security Analysis  
December 2016



# Outline

## Introduction

- Some Key Exchange Protocols
- Reductionist Security

## Key Exchange Models Before eCK

- Bellare–Rogaway Model Evolution
- Canetti–Krawczyk Model Evolution

## eCK Model and Beyond

- eCK Model
- Forward Secrecy
- Models including Functional Queries

## Summary and Conclusion

# Diffie–Hellman key exchange

**A**

$$r_A \in_R \mathbb{Z}_q$$

$$t_A = g^{r_A}$$

$$\xrightarrow{t_A}$$

**B**

$$r_B \in_R \mathbb{Z}_q$$

$$t_B = g^{r_B}$$

$$\xleftarrow{t_B}$$

$$Z_{AB} = t_B^{r_A}$$

$$Z_{BA} = t_A^{r_B}$$

- $r_A$  and  $r_B$  are *ephemeral secrets*
- $Z_{AB}$  is the *shared secret*



# HMQRV protocol

**A**

$$r_A \in_R \mathbb{Z}_q$$

$$t_A = g^{r_A}$$

$$S_A = r_A + dx_A \text{ mod } q$$

$$Z_{AB} = (t_B y_B^e)^{S_A}$$

**B**

$$r_B \in_R \mathbb{Z}_q$$

$$t_B = g^{r_B}$$

$$S_B = r_B + ex_B \text{ mod } q$$

$$Z_{BA} = (t_A y_A^d)^{S_B}$$

$$\xrightarrow{t_A}$$

$$\xleftarrow{t_B}$$

- $r_A$  and  $r_B$  are *ephemeral secrets*
- $x_A$  and  $x_B$  are *long-term secrets*
- $y_A = g^{x_A}$  and  $y_B = g^{x_B}$  are *public keys*
- $d = H(t_A, ID_B)$ ,  $e = H(t_B, ID_A)$



# NAXOS protocol

**A**

$$r_A \in_R \mathbb{Z}_q$$
$$h_A = H_1(x_A, r_A)$$

$$t_A = g^{h_A}$$

 $\xrightarrow{t_A}$ **B**

$$r_B \in_R \mathbb{Z}_q$$
$$h_B = H_1(x_B, r_B)$$

$$t_B = g^{h_B}$$

 $\xleftarrow{t_B}$ 

$$K_{AB} =$$
$$H_2(t_B^{x_A}, y_B^{h_A}, t_B^{h_A}, ID_A, ID_B)$$

$$K_{AB} =$$
$$H_2(y_A^{h_B}, t_A^{x_B}, t_A^{h_B}, ID_A, ID_B)$$

- $r_A$  and  $r_B$  are *ephemeral secrets*
- $x_A$  and  $x_B$  are *long-term secrets*
- $y_A = g^{x_A}$  and  $y_B = g^{x_B}$  are *public keys*
- $K_{AB}$  is the *session key*



## Jeong–Katz–Lee protocol TS3

**A**

$$r_A \in_R \mathbb{Z}_q$$

$$t_A = g^{r_A}$$

$$Z_{AB} = t_B^{r_A}$$

$$t_A, \text{MAC}_{K_M}(ID_A, ID_B, t_A) \xrightarrow{\hspace{2cm}}$$

$$t_B, \text{MAC}_{K_M}(ID_B, ID_A, t_B) \xleftarrow{\hspace{2cm}}$$

**B**

$$r_B \in_R \mathbb{Z}_q$$

$$t_B = g^{r_B}$$

$$Z_{BA} = t_A^{r_B}$$

- $r_A$  and  $r_B$  are *ephemeral secrets*
- $x_A$  and  $x_B$  are *long-term secrets*
- $y_A = g^{x_A}$  and  $y_B = g^{x_B}$  are *public keys*
- $K_M$  is MAC key derived from static Diffie–Hellman  $g^{x_A x_B}$



## Need for formal modelling

- History of failed protocol designs in 1980s and 1990s
- What is a valid attack?
- Obtain proofs of security
- Analysis of real world protocols

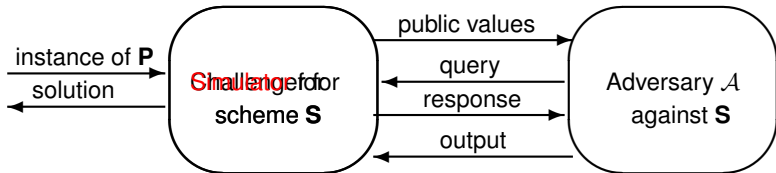


## Security games

- Adversary is an efficient probabilistic algorithm
- Challenger presents to the adversary  $\mathcal{A}$  any elements required by the model.
- Adversary's queries must be answered by the challenger as if the adversary is interacting with the protocol
- The game ends when the adversary halts its computations and gives its output
- A winning condition at the end of the game decides whether or not the adversary has won the game
- Security is defined based on the success of the adversary in the security game



## Security game for scheme $S$



- A successful adversary  $\mathcal{A}$  against scheme  $S$  can be used to construct a solution to an instance of problem  $P$
- We *reduce* the security of scheme  $S$  to the difficulty of problem  $P$



## Main elements in reductionist security analysis

- The security definition, including the specification of the challenger and a winning condition
- the specification of the protocol to be analysed
- a theorem and its proof bounding the probability that an adversary can win the security game in terms of some computational assumptions



## Game hopping

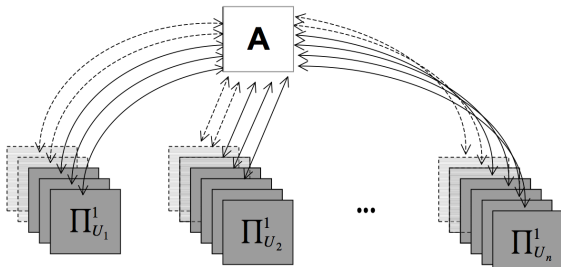
- A proof technique suitable for reductionist security proofs
- Security game *evolves* by changing rules for challenger
- Start with normal security game
- Each hop defines new game with quantifiable difference in adversary advantage
- End with game where adversary has zero advantage
- Typical game hops are:
  - changing distributions which the adversary cannot distinguish
  - aborting game if a certain event occurs



## Bellare and Rogaway's security model

- First computational model, ACM CCS 1993
- Adversary controls the security game by querying a set of *sessions* at a party
- A session  $\Pi_U^s$  represents the actions of party  $U$  in the protocol run indexed by integer  $s$
- Long-term keys are initialised using a key generation algorithm

- The adversary  $A$  is computationally bounded to probabilistic polynomial time





## Adversarial queries in BR model

<b>Query</b>	<b>Inputs</b>	<b>Outputs</b>
send	session + input message	output message
reveal	session	accepted session key
corrupt	party	long-term key
test	fresh session	session key / random

- To win the security game the adversary must correctly decide the bit used in the answer to the test query



## Freshness

- The test query may only be used for a *fresh* session
- An session is said to be *fresh* when:
  - it has accepted a session key, and
  - neither itself nor its *partner* have had a corrupt or reveal query
- The way of defining partners has varied in different models
- Original BR93 model defines partners to be sessions with *matching conversations*

## BR model versions

<b>Model</b>	<b>Setting</b>	<b>Partnering mechanism</b>
BR93	2-party shared key	Matching conversations
BR95	Server-based	Partner function
SR96	Smart card	Partner function
BWM97	Public key	Matching conversations
BWJM97	Key agreement	Matching conversations
BPR00	Password-based	Session identifiers



## Which elements are available to the adversary?

- Different models allow different combinations

Actor	owner of the test session
Peer	(intended) partner of the test session
✓	element is available (leaked or chosen)
(✓)	element may be available
$\mathcal{F}$	a (restricted) function of the element is available

- Table shows only test session
- Usually all elements are available for non-test sessions



## Which elements are available to the adversary?

— BR model

	Before test session	After test session
Actor long-term		
Actor ephemeral		
Peer long-term		
Peer ephemeral	(✓)	

For some protocols (such as HMQV) an active adversary can choose ephemeral key of peer session



## Modelling forward secrecy

- A protocol provides forward secrecy if adversary cannot distinguish session key from a random string even given the long-term keys after test session is complete
- Allow adversary to obtain long-term keys, *after* test session is complete
- Widely seen as desirable real-world property today
- Introduces *timing* into the model



## Which elements are available to the adversary?

- BPR00 model with forward secrecy

	Before test session	After test session
Actor long-term		✓
Actor ephemeral		
Peer long-term		✓
Peer ephemeral	(✓)	

- Which protocols provide forward secrecy?

## Canetti–Krawczyk (CK01) model

- Similar basic idea to Bellare–Rogaway models
- Two main motivations:
  - build secure channels for sessions
  - a modular design approach using *authenticators*
- Allows *session state* to be revealed



## HMQV model

- Enhancement of CK01 model used to analyse HMQV protocol
- Session state query reveals ephemeral private key
- Key compromise impersonation (KCI) attacks are captured by allowing adversary to obtain private key of the owner of the test session



## Which elements are available to the adversary?

- HMQV model capturing KCI attack

	Before test session	After test session
Actor long-term	✓	✓
Actor ephemeral		
Peer long-term		
Peer ephemeral	(✓)	

- Which protocols provide KCI resistance?



## Common elements of all models

- Adversary controls network
- Some mechanism identifies partners of sessions
- Adversary can obtain session key from sessions other than test session and its partner (if it exists)
- Adversary wins by distinguishing session key of test session from random string





## The eCK model

- Proposed at Provsec 2007 by LaMacchia, Lauter and Mityagin, now widely referred to as eCK model
- Tackles directly some limitations in the CK and BR models. Specific advantages are:
  - the adversary can obtain ephemeral secrets which belong to the test session;
  - the adversary can obtain the long-term key of the test session and of its partner even before the session is completed.

## Which elements are available to the adversary?

— eCK model

	Before test session	After test session
Actor long-term	✓	✓
Actor ephemeral		
Peer long-term		
Peer ephemeral	✓	✓

or

Actor long-term		
Actor ephemeral	✓	✓
Peer long-term		
Peer ephemeral	✓	✓

or ...



## Which elements are available to the adversary?

- eCK model if adversary is passive in test session

	Before test session	After test session
Actor long-term	✓	✓
Actor ephemeral		
Peer long-term	✓	✓
Peer ephemeral		

or

Actor long-term		
Actor ephemeral	✓	✓
Peer long-term	✓	✓
Peer ephemeral		

- NAXOS protocol is secure in eCK model

## Strong and weak forward secrecy

### Strong forward secrecy (sFS)

- Adversary takes an active part in the session under attack
- Victim executes session with the adversary

### Weak forward secrecy (wFS)

- Adversary is prevented from taking an active part in the session under attack
  - Victim executes the session with a legitimate party
- 
- eCK model *cannot* capture strong forward secrecy since it does not consider timing

## Which elements are available to the adversary?

— eCK-PFS (Cremers–Feltz, 2012)

	Before test session	After test session
Actor long-term	✓	✓
Actor ephemeral		
Peer long-term		✓
Peer ephemeral	✓	✓

or,

	Before test session	After test session
Actor long-term	✓	✓
Actor ephemeral		
Peer long-term	✓	✓
Peer ephemeral		

or ...



## Leakage resilient key exchange

- Aims to capture side channel attacks
- Adversary gets access to a chosen function of the long-term secret with some restrictions
  - Leakage can be continuous or bounded
  - Leakage can be restricted to before the test session occurs
- First results by Moriyama and Okamoto, 2011 – assume *before-the-fact* leakage
- ASB 2015 achieve *continuous, after the fact leakage* (CAFL) security in an eCK type model

## Which elements are available to the adversary?

- Leakage resilient model (CAFL-eCK)

	Before test session	After test session
Actor long-term	✓	✓
Actor ephemeral		
Peer long-term	$\mathcal{F}$	$\mathcal{F}$
Peer ephemeral	✓	✓

or

Actor long-term	$\mathcal{F}$	$\mathcal{F}$
Actor ephemeral	✓	✓
Peer long-term	✓	✓
Peer ephemeral		

or ...

- $\mathcal{F}$  is restricted function of long-term secret



## Post-compromise security

- Analysed by Cohn-Gordon, Cremers and Garratt, IEEE Security and Privacy 2016
- Adversary can obtain (partial) information about long-term key *before* test session
- Models temporary loss of long-term secrets





## Which elements are available to the adversary?

- Post-compromise security - weak compromise

	Before test session	After test session
Actor long-term	✓	✓
Actor ephemeral		
Peer long-term	$\mathcal{F}$	
Peer ephemeral	✓	✓

- $\mathcal{F}$  is interface to long-term secret, for example *HSM*
- $\mathcal{F}$  queries can be added to adversary queries for test session before completed
- Seems similar to CAFL-eCK but restrictions on  $\mathcal{F}$  are different



## Which elements are available to the adversary?

- Post-compromise security - full compromise

	Before test session	After test session
Actor long-term	✓	✓
Actor ephemeral		
Peer long-term	✓ then ✗	✓
Peer ephemeral		

- Can only be satisfied using *stateful* protocols
- Long-term keys evolve over time (ratcheting)



## Which elements are available to the adversary?

- Mass surveillance model?

	Before test session	After test session
Actor long-term		✓
Actor ephemeral		✓
Peer long-term		✓
Peer ephemeral		✓

- Adversary is passive before test session
- Adversary can learn secrets after test session
- No stateless protocol is secure in this model



## Which elements are available to the adversary?

- Weaker mass surveillance model?

	Before test session	After test session
Actor long-term		
Actor ephemeral		✓
Peer long-term		
Peer ephemeral		✓

- Adversary is passive before test session
- Adversary can learn secrets after test session
- **No TLS 1.2 or TLS 1.3 variant is secure in this model**



## Introduction

- Some Key Exchange Protocols
- Reductionist Security

## Key Exchange Models Before eCK

- Bellare–Rogaway Model Evolution
- Canetti–Krawczyk Model Evolution

## eCK Model and Beyond

- eCK Model
- Forward Secrecy
- Models including Functional Queries

## Summary and Conclusion



## Current and future challenges

- Is indistinguishability the right definition for real-world key exchange?
- Security against ephemeral key leakage for real-world protocols
- Post-quantum security
- Taming complexity ... with automation?
- Classifying and unifying models ... stateful protocols, functional security, ...
- More real-world protocols: DTLS, ZRTP, ...
- Modelling humans
- All of the above in the group setting



## Key References

- Alex Dent, An Brief Introduction to Provable Security (Slides), <http://www.cs.bris.ac.uk/Research/CryptographySecurity/SummerSchool2009/slides/Alex.pdf>
- Douglas Stebila, An Introduction to Provable Security (Slides), <https://www.douglas.stebila.ca/files/teaching/amsi-winter-school/Lecture-2-3-Provable-security.pdf>
- Victor Shoup, Sequences of games: a tool for taming complexity in security proofs. IACR Cryptology ePrint Archive 2004/11/30
- Entity authentication and key distribution, M Bellare, P Rogaway - ACM CCS 1993
- Analysis of key-exchange protocols and their use for building secure channels, R Canetti, H Krawczyk - Crypto 2001



## Key References

- Stronger security of authenticated key exchange, B LaMacchia, K Lauter, A Mityagin - International Conference on Provable Security, 2007
- On forward secrecy in one-round key exchange, C Boyd, JG Nieto - IMA Conference on Cryptography and Coding, 2011
- Leakage resilient eCK-secure key exchange protocol without random oracles D Moriyama and T Okamoto, ASIACCS 2011
- Beyond eCK: perfect forward secrecy under actor compromise and ephemeral-key reveal, C Cremers, M Feltz - Designs, Codes and Cryptography, 2015
- Continuous after-the-fact leakage-resilient eCK-secure key exchange, J Alawatugoda, D Stebila, C Boyd - IMA Conference on Cryptography and Coding, 2015
- On Post-Compromise Security, K Cohn-Gordon, C Cremers, L Garratt - IEEE Security and Privacy Symposium 2016



## **Modelling the Security of Key Exchange**

Colin Boyd

including joint work with Janaka Alawatugoda,  
Juan Gonzalez Nieto

Department of Telematics, NTNU

Workshop on Tools and Techniques for Security Analysis  
December 2016